Listy Biometryczne - Biometrical Letters
Vol. 31 (1994), No. 1, 17-25

Reinforced block designs with two groups of treatments

B. Ceranka, J. Krzyszkowska

Department of Mathematical and Statistical Methods
Agricultural University of Poznań, Poland

Summary

The paper presents construction of some types of designs with two groups of treatments. Spectral forms of information matrices are used.

1. Introduction

We consider different types of augmented block designs with two groups of treatments; we divide v treatments into two groups of v_1 and v_2 treatments ($v = v_1 + v_2$). The treatments of the first group are often called the standard or basic ones and the treatments of the second group are called the additional or control ones (see e.g. Das, 1958; Federer, 1961; Pearce,1960). The general scheme and the incidence matrix of the above characterized block design have the form, respectively:

d_1
d_2

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \end{bmatrix}$$

where $d_1(v_1, b_1, r_1, \mathbf{k}_1)$ is a block design for the basic treatments and $d_2(v_2, b_1, r_2, \mathbf{k}_2)$ is a block design for the additional treatments.

Moreover, similarly as in the work of Ceranka and Krzyszkowska (1992), we add to the design some additional blocks, so in both groups of treatments it is necessary to reserve a constant number of replications. The obtained design d is

Key words: group divisible partially balanced incomplete block design with two associate classes

described uniquely by the scheme and the incidence matrix of the form, respectively:

d_1	d_3
d_2	d_4

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_3 \\ \mathbf{N}_2 & \mathbf{N}_4 \end{bmatrix}, \tag{1.1}$$

where $\mathbf{N}_1$, $\mathbf{N}_3$ and $\mathbf{N}_2$, $\mathbf{N}_4$ are the incidence matrices of the basic and additional designs, respectively, and $r_1 v_1 = \mathbf{k}'_1 \mathbf{1}_{b_1} = n_1$, $r_2 v_2 = \mathbf{k}'_2 \mathbf{1}_{b_1} = n_2$, $r_3 v_1 = \mathbf{k}'_3 \mathbf{1}_{b_2} = n_3$, $r_4 v_2 = \mathbf{k}'_4 \mathbf{1}_{b_2} = n_4$, $n_1 + n_2 + n_3 + n_4 = n$.

The parameters of design d are equal to: $v = v_1 + v_2$, $b = b_1 + b_2$, $\mathbf{r} = ((r_1 + r_3)\mathbf{1}'_{v_1} \vdots (r_2 + r_4)\mathbf{1}'_{v_2})'$, $\mathbf{k} = ((\mathbf{k}_1 + \mathbf{k}_2)' \vdots (\mathbf{k}_3 + \mathbf{k}_4)')'$, where:

v is the number of treatments,

b is the number of blocks,

$\mathbf{r} = (r_1,\ldots, r_v)'$ is the vector of replications,

$\mathbf{k} = (k_1,\ldots k_b)'$ is the vector of block sizes,

and some special matrices and vectors are:

$\mathbf{1}_i$ is the $i \times 1$ vector with every element equal to unity,

$\mathbf{I}_i$ is the $i \times i$ identity matrix,

$\mathbf{r}'$ is the transpose of $\mathbf{r}$,

$\mathbf{r}^{\delta} = \mathrm{diag}(r_1,\ldots, r_v)$ is diagonal matrix whose diagonal elements are those of the vector $\mathbf{r}$,

$\mathbf{r}^{-\delta}$ is the inverse of $\mathbf{r}^{\delta}$.

Let us note that both the basic and the additional treatments are replicated in these blocks in such a way that the requirement of the constant number of replications is preserved for both groups of treatments.

2. Construction of reinforced block design

For the estimation of treatment comparisons the matrix of the reduced normal equations, the so-called information matrix $\mathbf{C}$, is used:

$$\mathbf{C} = \mathbf{r}^{\delta} - \mathbf{N}\mathbf{k}^{-\delta}\mathbf{N}' .$$

If the information matrix for the design with incidence matrix $\mathbf{N}$ is denoted by $\mathbf{C}(\mathbf{N})$, then we will notice that:

$$\mathbf{C}(\mathbf{N}) = \mathbf{C}\left(\begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \end{bmatrix}\right) + \mathbf{C}\left(\begin{bmatrix} \mathbf{N}_3 \\ \mathbf{N}_4 \end{bmatrix}\right) . \tag{2.1}$$

The property (2.1) makes easy the analysis of the reinforced designs (i.e. the designs with additional blocks).

I. Let the design d_1 be the group divisible partially balanced incomplete block design with two associate classes (GDPBIB(2)) with parameters $v_1 = ms$, $b_1, r_1, k_1, \lambda_1, \lambda_2$, i.e. the design in which ms treatments can be divided into m groups of s treatments each. The design is such that all pairs of treatments belonging to the same group occur together in λ_1 blocks, while pairs of treatments from different groups occur together in λ_2 blocks. Let us notice that λ_1 and λ_2 are elements of the association matrix $\mathbf{N}_1\mathbf{N}_1'$ (see John, 1987).

The association matrix for this design can be presented in the spectral decomposition

$$\mathbf{N}_1\mathbf{N}_1' = \rho_0\mathbf{X}_0 + \rho_1\mathbf{X}_1 + \rho_2\mathbf{X}_2 ,$$

where $\rho_0 = r_1k_1$, $\rho_1 = r_1 - \lambda_1$, $\rho_2 = r_1k_1 - v_1\lambda_2$ are eigenvalues of this matrix with multiplicities $1, m(s-1), m-1$, respectively while the matrices $\mathbf{X}_0$, $\mathbf{X}_1$, $\mathbf{X}_2$ have the following forms

$$\begin{aligned} \mathbf{X}_0 &= \mathbf{1}_{v_1}\mathbf{1}_{v_1}'/v_1 , \\ \mathbf{X}_1 &= \mathbf{I}_m \otimes (\mathbf{I}_s - \mathbf{1}_s\mathbf{1}_s'/s) , \\ \mathbf{X}_2 &= (\mathbf{I}_m - \mathbf{1}_m\mathbf{1}_m'/m) \otimes (\mathbf{1}_s\mathbf{1}_s'/s) , \end{aligned}$$

where $\otimes$ denotes the Kronecker product of matrices. Other elementary explanations can be found in the work of Bogacka et al. (1990).

Let d_2 be a randomized complete block design. The matrix $\mathbf{C}$ for the design with the incidence matrix

$$\begin{bmatrix} \mathbf{N}_1 \\ \mathbf{1}_{v_2}\mathbf{1}_{b_1}' \end{bmatrix}$$

and parameters v, b_1, $\mathbf{r} = (r_1\mathbf{1}_{v_1}' \vdots b_1\mathbf{1}_{v_2}')'$, $\mathbf{k} = (k_1 + v_2)\mathbf{1}_{b_1}$ has the following form:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} ,$$

where

$$\begin{aligned} \mathbf{C}_{11} &= \sum_{i=0}^{2} (r_1 - \rho_i/(k_1 + v_2))\mathbf{X}_i , \\ \mathbf{C}_{12} &= -(r_1/(k_1 + v_2))\mathbf{1}_{v_1}\mathbf{1}_{v_2}' , \\ \mathbf{C}_{21} &= -(r_1/(k_1 + v_2))\mathbf{1}_{v_2}\mathbf{1}_{v_1}' , \\ \mathbf{C}_{22} &= b_1\mathbf{I}_{v_2} - (b_1/(k_1 + v_2))\mathbf{1}_{v_2}\mathbf{1}_{v_2}' . \end{aligned}$$

The basic contrasts between treatments can be divided into four groups. In the first and the second group, both concerning basic treatments, there are contrasts relevant to eigenvalues ρ_1 and ρ_2, respectively. The third group contains the comparisons of additional treatments, and the fourth group consists of the single contrast which is a comparison of the group of basic treatments with the group of additional treatments. In particular groups there are $m(s-1)$, $m-1$, v_2-1, 1 contrasts.

These groups are generated by the matrices $\mathbf{L}_1$, $\mathbf{L}_2$, $\mathbf{L}_3$, $\mathbf{L}_4$ of the form:

$$\mathbf{L}_1 = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{L}_2 = \begin{bmatrix} \mathbf{X}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{L}_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{v_2} - \mathbf{1}_{v_2}\mathbf{1}'_{v_2}/v_2 \end{bmatrix},$$

$$\mathbf{L}_4 = \frac{1}{v_1 v_2 v}\begin{bmatrix} v_2^2\mathbf{1}_{v_1}\mathbf{1}'_{v_1} & -v_1v_2\mathbf{1}_{v_1}\mathbf{1}'_{v_2} \\ -v_1v_2\mathbf{1}_{v_2}\mathbf{1}'_{v_1} & v_1^2\mathbf{1}_{v_2}\mathbf{1}'_{v_2} \end{bmatrix}. \tag{2.2}$$

The ranks of these matrices are equal to:

$$r(\mathbf{L}_1) = m(s-1), \quad r(\mathbf{L}_2) = m-1, \quad r(\mathbf{L}_3) = v_2-1, \quad r(\mathbf{L}_4) = 1 .$$

Moreover, these matrices fulfil the condition $\mathbf{CL}_i = \mu_i\mathbf{L}_i$ for i=1,2,3,4, where $\mu_i = r_1-\rho_1/(k_1+v_2)$ with multiplicity $r(\mathbf{L}_1)$, $\mu_2 = r_1-\rho_2/(k_1+v_2)$ with multiplicity $r(\mathbf{L}_2)$, $\mu_3 = b_1$ with multiplicity $r(\mathbf{L}_3)$, $\mu_4 = r_1 v/(k_1+v_2)$ with multiplicity $r(\mathbf{L}_4)$.

From the above and from equality $\sum_{i=0}^{4}\mathbf{L}_i = \mathbf{I}_v$, there follows the possibility of writing the information matrix for this design in the form:

$$\mathbf{C}\left(\begin{bmatrix} \mathbf{N}_1 \\ \mathbf{1}_{v_2}\mathbf{1}'_{b_1} \end{bmatrix}\right) = \sum_{i=0}^{4}\mu_i\mathbf{L}_i , \tag{2.3}$$

where $\mu_0 = 0$, $\mathbf{L}_0 = \mathbf{1}_v\mathbf{L}'_v/v$. $\mathbf{L}_i$ are mutually orthogonal idempotent matrices. From equality (2.3) it follows that μ_i are eigenvalues of the matrix $\mathbf{C}$.

In a particular case, when $\mathbf{N}_2 = \mathbf{1}'_{b_1}$, we can get the design analyzed in the work of Bogacka et al. (1990). Brzeskwiniewicz (1993) considered more general case, when d_1 is partially balanced incomplete block designs.

Now, let us expand the design d by additional blocks, getting the design with the incidence matrix:

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_3 \\ \mathbf{1}_{v_2}\mathbf{1}'_{b_1} & \mathbf{N}_4 \end{bmatrix},$$

where $\mathbf{N}_1$ is (as above) the incidence matrix for GDPBIB(2). In order to determine the matrix $\mathbf{C}$ we can use (2.1). So, it is enough to set the matrix:

$$\mathbf{C}(\begin{bmatrix}\mathbf{N}_3\\ \mathbf{N}_4\end{bmatrix}) . \tag{2.4}$$

Let us consider some cases.

Case 1.1. Let d_3 be a balanced incomplete block design with parameters $v_1, b_2, r_3, k_3, \lambda_3$ and let d_4 be a randomized complete block design, i.e. $\mathbf{N}_4 = \mathbf{1}_{v_2}\mathbf{1}'_{b_2}$. Since d_3 is particular case of d_1 ($\rho_0 = r_3k_3$, $\rho_1 = \rho_2 = r_3 - \lambda_3$) then eigenvalues of design

d_3
d_4

are equal to: $\nu_1 = \nu_2 = r_3 - (r_3 - \lambda_3)/(k_3 + v_2)$ with multiplicity v_1–1, $\nu_3 = b_2$ with multiplicity v_2–1, $\nu_4 = r_3v/(k_3 + v_2)$ with multiplicity 1. The matrix (2.3) can also be presented in spectral form:

$$\mathbf{C}(\begin{bmatrix}\mathbf{N}_3\\ \mathbf{N}_4\end{bmatrix}) = \sum_{i=1}^{3} \nu_i\mathbf{L}_i .$$

Then, the eigenvalues of design d are sums of eigenvalues of designs

d_1
d_2

and

d_3
d_4

and also:

$\mu_1 + \nu_1 = r_1 - \rho_1/(k_1 + v_2) + r_3 - (r_3 - \lambda_3)/(k_3 + v_2)$ with multiplicity $m(s-1)$,

$\mu_2 + \nu_2 = r_1 - \rho_2/(k_1 + v_2) + r_3 - (r_3 - \lambda_3)/(k_3 + v_2)$ with multiplicity $m-1$,

$\mu_3 + \nu_3 = b_1 + b_2$ with multiplicity $v_2 - 1$,

$\mu_4 + \nu_4 = r_1v/(k_1 + v_2) + r_3v/(k_3 + v_2)$ with multiplicity 1.

Case 1.2. Let $\begin{bmatrix}\mathbf{N}_3\\ \mathbf{N}_4\end{bmatrix} = \tilde{\mathbf{N}}$ be the incidence matrix for a balanced incomplete block design with parameters v, b_2, r_3, k_3. This design has only one non zero eigenvalue $\nu = \lambda_3v/k_3$ and the eigenvalues of design d are equal to:

$\mu_1 + \nu = r_1 - \rho_1/(k_1 + v_2) + \lambda_3v/k_3$ with multiplicity $m(s-1)$,

$\mu_2 + \nu = r_1 - \rho_2/(k_1 + v_2) + \lambda_3v/k_3$ with multiplicity $m-1$,

$\mu_3 + \nu = b_1 + \lambda_3 v / k_3$ with multiplicity $v_2 - 1$,
$\mu_4 + \nu = r_1 v / (k_1 + v_2) + \lambda_3 v / k_3$ with multiplicity 1.

In particular case, if $\tilde{\mathbf{N}} = \mathbf{1}_v \mathbf{1}'_{b_2}$ then $\nu = b_2$.

Case 1.3. Let $\mathbf{N}_3 = \mathbf{1}_{v_1} \mathbf{1}'_{b_2}$, and let $\mathbf{N}_4$ be the incidence matrix for a balanced incomplete block design. The eigenvalues of design d of form (1.1) are equal to:

$\mu_1 + \nu_1 = r_1 - \rho_1 / (k_1 + v_2) + b_2$ with multiplicity $m(s-1)$,
$\mu_2 + \nu_2 = r_1 - \rho_2 / (k_1 + v_2) + b_2$ with multiplicity $m-1$,
$\mu_3 + \nu_3 = b_1 + r_4 - (r_r - \lambda_4) / (k_4 + v_2)$ with multiplicity $v_2 - 1$,
$\mu_4 + \nu_4 = r_1 v / (k_1 + v_2) + r_4 v / (k_4 + v_2)$ with multiplicity 1.

II. Let d_1 and d_2 be the balanced incomplete block designs with parameters $d_1(v_1, b_1, r_1, k_1)$ and $d_2(v_2, b_1, r_2, k_2)$, and let they balance each other, which means that any basic treatment from design d_1 occurs together with any additional treatment from design d_2 in $\lambda_{12} = \lambda_{21}$ blocks (see e.g. Corsten, 1962). At the same time, the following conditions are true:

$$\mathbf{N}_1 \mathbf{N}'_2 = \lambda_{12} \mathbf{1}_{v_1} \mathbf{1}'_{v_2}, \quad r_1 k_2 = \lambda_{12} v_2, \quad r_2 k_1 = \lambda_{12} v_1 .$$

As in the point I we can use formula (2.1) to determine the incidence matrix for design d. We can see that the information matrix for design

d_1
d_2

can be presented in spectral form:

$$\mathbf{C}\left(\begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \end{bmatrix}\right) = \sum_{i=0}^{3} \mu_i \mathbf{L}_i ,$$

where

$$\mathbf{L}_0 = \mathbf{1}_v \mathbf{1}'_v / v , \qquad \mathbf{L}_1 = \begin{bmatrix} \mathbf{I}_v - \mathbf{1}_{v_1} \mathbf{1}'_{v_1} / v_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{L}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{v_2} - \mathbf{1}_{v_2} \mathbf{1}'_{v_2} / v_2 \end{bmatrix},$$

$$\mathbf{L}_3 = \frac{1}{v_1 v_2 v} \begin{bmatrix} v_2^2 \mathbf{1}_{v_1} \mathbf{1}'_{v_1} & -v_1 v_2 \mathbf{1}_{v_1} \mathbf{1}'_{v_2} \\ -v_1 v_2 \mathbf{1}_{v_2} \mathbf{1}'_{v_1} & v_1^2 \mathbf{1}_{v_2} \mathbf{1}'_{v_2} \end{bmatrix},$$

and where $\mu_0 = 0$, $\mu_1 = r_1 - (r_1 - \lambda_1) / (k_1 + k_2)$, $\mu_2 = r_2 - (r_2 - \lambda_2) / (k_1 + k_2)$, $\mu_3 = \lambda_{12} v / (k_1 + k_2)$ with multiplicities 1, $v_1 - 1$, $v_2 - 1$, 1, respectively.

We can consider here the same cases as in point I.

Case 2.1. Let $\mathbf{N}_3$ be the incidence matrix for a balanced incomplete block design and $\mathbf{N}_4 = \mathbf{1}_{v_2}\mathbf{1}'_{b_2}$. The eigenvalues of the design d are equal to:

$\mu_1 + \nu_1 = r_1 - (r_1 - \lambda_1)/(k_1 + k_2) + r_3 - (r_3 - \lambda_3)/(k_3 + v_2)$ with multiplicity $v_1 - 1$,
$\mu_2 + \nu_2 = r_2 - (r_2 - \lambda_2)/(k_1 + k_2) + b_2$ with multiplicity $v_2 - 1$,
$\mu_3 + \nu_3 = \lambda_{12}v\ /(k_1 + k_2) + r_3 v/(k_3 + v_2)$ with multiplicity 1.

Case 2.2. Let $\begin{bmatrix}\mathbf{N}_3\\ \mathbf{N}_4\end{bmatrix} = \tilde{\mathbf{N}}$ be the incidence matrix for a balanced incomplete block design with parameters $v, b_2, r_3, k_3, \lambda_3$. Then this design has only one non-zero eigenvalue $\nu = \lambda_3 v\ /k_3$ and the eigenvalues of design d are equal to:

$\mu_1 + \nu = r_1 - (r_1 - \lambda_1)/(k_1 + k_2) + \lambda_3 v/k_3$ with multiplicity $v_1 - 1$,
$\mu_2 + \nu = r_2 - (r_2 - \lambda_2)/(k_1 + k_2) + \lambda_3 v/k_3$ with multiplicity $v_2 - 1$,
$\mu_3 + \nu = \lambda_{12}v\ /(k_1 + k_2) + \lambda_3 v\ /k_3$ with multiplicity 1.

Case 2.3. Let $\mathbf{N}_3 = \mathbf{1}_{v_1}\mathbf{1}'_{b_2}$, and let $\mathbf{N}_4$ be the incidence matrix for a balanced incomplete block design. The eigenvalues of design d of form (1.1) are equal to:

$\mu_1 + \nu_1 = r_1 - (r_1 - \lambda_1)/(k_1 + k_2) + b_2$ with multiplicity $v_1 - 1$,
$\mu_2 + \nu_2 = r_2 - (r_2 - \lambda_2)/(k_1 + k_2) + r_4 - (r_4 - \lambda_4)/(k_4 + v_2)$ with multiplicity $v_2 - 1$,
$\mu_3 + \nu_3 = \lambda_{12}v\ /(k_1 + k_2) + r_4 v\ /(k_4 + v_2)$ with multiplicity 1.

3. Example

The GDPBIB(2) is, among others, used in two-factor experiments. Let us assume that factorials A and B have m and s levels, respectively. We do combinations of these factors according to the following scheme

$$\begin{array}{cccc} A_1B_1, & A_1B_2, & \ldots, & A_1B_s, \\ A_2B_1, & A_2B_2, & \ldots, & A_2B_s, \\ \ldots & \ldots & \ldots & \ldots \\ A_mB_1, & A_mB_2, & \ldots, & A_mB_s. \end{array}$$

We can apply the levels of factorial A to groups.

Let d_1 be the GDPBIB(2) with parameters: $v_1 = 6$, $b_1 = 3$, $r_1 = 2$, $k_1 = 4$, $\lambda_1 = 2$, $\lambda_2 = 1$, $m = 3$, $s = 2$; where the columns are the blocks

$$\begin{array}{ccc} 1 & 1 & 3 \\ 2 & 2 & 4 \\ 3 & 5 & 5 \\ 4 & 6 & 6 \end{array}\ .$$

The association scheme is

1st group: 1, 2
2nd group: 3, 4
3rd group: 5, 6.

For instance, treatments 1 and 2 are first associates and occur together in the first and second block, while treatments 1 and 3 are second associates and occur together only in the first block.

Let d_1 be the GDPBIB(2) (with parameters as above), d_2 be a randomized complete block design with $\mathbf{N}_2 = \mathbf{1}_2\mathbf{1}'_3$, d_3 be a balanced incomplete block design with parameters $v_1 = 6$, $b_2 = 10$, $r_3 = 5$, $k_3 = 3$, $\lambda_3 = 2$ and d_4 be a randomized complete block design with $\mathbf{N}_4 = \mathbf{1}_2\mathbf{1}'_{10}$. Then the eigenvalues of the matrix $\mathbf{C}$ are: $\mu_1 + \nu_1 = 32/5$, $\mu_2 + \nu_2 = 91/15$, $\mu_3 + \nu_3 = 13$, $\mu_4 + \nu_4 = 32/3$.

The matrices $\mathbf{L}_i$ are of the form (2.2). Therefore, for matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}$$

we have:

$$\mathbf{C}_{11} = (32/5)\mathbf{I}_6 - (1/6)\mathbf{I}_3 \otimes \mathbf{1}_2\mathbf{1}'_2 - (17/30)\mathbf{1}_6\mathbf{1}'_6 ,$$

$$\mathbf{C}_{12} = (-4/3)\mathbf{1}_6\mathbf{1}'_2 ,$$

$$\mathbf{C}_{21} = (-4/3)\mathbf{1}_2\mathbf{1}'_6 ,$$

$$\mathbf{C}_{22} = 13\mathbf{I}_2 - (5/2)\mathbf{1}_2\mathbf{1}'_2 .$$

REFERENCES

Bogacka B., Krzyszkowska J., Mejza S. (1990). Układy blokowe o grupach podzielnych rozszerzone o obiekt kontrolny. *Dwudzieste Colloquium Metodologiczne z Agrobiometrii*, PAN, 135-146.

Brzeskwiniewicz H. (1993). Analysis of partially balanced incomplete block designs with standards. *Biometrical Journal* **35**, 407-417.

Ceranka B., Krzyszkowska J. (1992). Block designs with two groups of treatments. *Listy Biometryczne - Biometrical Letters* **29**, 33-44.

Corsten L.C. (1962). Balanced block designs with two different number of replicates. *Biometrics* **18**, 499-519.

Das M.N. (1958). On reinforced incomplete block designs. *Journal of Indian Society of Agricultural Statistics* **10**, 73-77.

Federer W.T. (1961). Augmented designs with one-way elimination of heterogeneity. *Biometrics* **17**, 447-473.

John J.A., (1987). *Cyclic designs*. Chapman and Hall, London.
Pearce S.C. (1960). Supplemented balance. *Biometrika* **47**, 263-271.

Received 2 October 1993; revised 1 February 1994

Wzmocnione układy bloków z dwiema grupami obiektów

Streszczenie

Praca zawiera konstrukcje pewnych typów układów z dwiema grupami obiektów. Wykorzystana jest postać spektralna macierzy informacji układu.

Słowa kluczowe: układ grup podzielonych o blokach niekompletnych z dwiema klasami partnerstwa